(Pages : 4) **4706**

**Reg. No. : ...............................**

**Name : ...................................**

# Fifth Semester B.Tech. Degree Examination, September 2014
## (2008 Scheme)
## (Special Supplementary)
## 08.502 : ADVANCED MATHEMATICS AND QUEUING MODELS (RF)

Time : 3 Hours Max. Marks : 100

***Instructions :*** *Answer **all** questions of Part **A** and **one full** question **each** from Module **I**, Module **II** and Module **III**.*

PART – A

1. Define slack and surplus variables. Explain their role in LPP.

2. Rewrite in standard form the following LPP.

   Min $z = x_1 + 2x_2 - 4x_3$

   Subject to $2x_1 + 2x_2 - x_3 \le 16$

   $x_1 - x_2 + x_3 = 8$

   $-x_1 + 2x_2 - x_3 \ge -7$

   $x_1 + x_3 \le 2$

   where $x_1$ , $x_3 \ge 0$ and $x_2$ is unrestricted in sign.



3. What are the three main phases of a project ?

4. Define the following :

   a) Most likely time b) Optimistic time

   c) Pessimistic time d) Expected time

5. Find the LU factorization of $\begin{bmatrix} 3 & -7 & -2 & 2 \\ -3 & 5 & 1 & 0 \\ 6 & -4 & 0 & -5 \\ -9 & 5 & -5 & 12 \end{bmatrix}$

P.T.O.

6. Show that the vectors $u_1 = (1, 1, 1)$, $u_2 = (1, 2, 3)$ and $u_3 = (2, 3, 8)$ span $\mathbb{R}^3$.

7. Find a least square solution of the system $\begin{bmatrix} -1 & 2 \\ -1 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

8. Explain (M/M/I) : (N/FIFO) queuing model with an example.

9. Find the probability that the queue size $\geq n$.

10. What do you understand by a queue ? Give some important applications of queuing theory. **(4×10=40 Marks)**

PART – B

**Module – I**

11. Solve the following LPP.

Minimize $z = 2x_1 + 9x_2 + x_3$ such that

$x_1 + 4x_2 + 2x_3 \geq 5,$

$3x_1 + x_2 + 2x_3 \geq 4,\ x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0.$

12. A small project is composed of seven activities whose time estimates are listed in the table as follows :

| **Activity :** | **(i – j):** | **(1 – 2)** | **(1 – 3)** | **(1 – 4)** | **(2 – 5)** | **(3 – 5)** | **(4 – 6)** | **(5 – 6)** |
|---|---|---|---|---|---|---|---|---|
| Opt. ($t_o$) : | t | 1 | 2 | 1 | 2 | 1 | 2 | 3 |
| Most likely $t_m$ : | t | 4 | 2 | 1 | 5 | 1 | 5 | 6 |
| Pessimistic $t_p$ : | 7 | 7 | 8 | 1 | 14 | 7 | 8 | 15 |

a) Draw the project network.

b) Calculate the length and variance of the critical path.

c) What is the probability that the project will be completed atleast four weeks earlier than expected.

d) If the project due date is 19 weeks what is the probability of meeting the due date.

## Module – II

13. a) Solve the following equation A X = B by using LU factorization of A.

$$x + 3y + 4z = 1$$

$$-3x - 6y - 7z + 2t = -2$$

$$3x + 3y - 4t = -1$$

$$-5x - 3y + 2z + 9t = 2$$

b) Find the dimension and construct a basis for

C(A), R(A), N(A) and N($A^T$) if $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$



c) Find an orthonormal basis for the subspace spanned by (1, –1, 0, 0), (0, 1, –1, 0) and (0, 0, 1, –1).

14. a) Find a spanning set for the null space of the matrix $A = \begin{bmatrix} 1 & -2 & 0 & 4 & 0 \\ 2 & -4 & 1 & -1 & 0 \\ 3 & -6 & 0 & 12 & 1 \end{bmatrix}$.

b) Find the singular value decomposition of $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$.

## Module – III

15. a) Consider a self service store with one cashier. Assume Poisson arrivals and exponential service time. Suppose that nine customers arrive on the average every 5 minutes and the cashier can serve 10 in 5 minutes. Find :

i) The average number of customers queuing for service.

ii) Average time a customer spends in the system.

iii) Average time a customer waits before being served.

iv) The probability of having more than 10 customers in the system.

v) The probability that a customer has to queue for more than 2 minutes.

b) If for a period of 2 hours in a day (8 – 10 AM) trains arrive at the yard every 20 minutes but the service time continues to remain 36 minutes, then calculate for this period

i) The probability that the yard is empty.

ii) Average queue length assuming that capacity of the yard is 4 trains only.

16. A general insurance company has 3 claim adjusters in its branch office. People with claims against the company are found to arrive in Poisson fashion at an average rate of 20 per 8 hour day. The amount of time that an adjuster spends with a claimant is found to be negative exponential distribution with mean service time 40 minutes. Claimants are processed in the order of their appearance.

a) How many hours a week can an adjuster expect to spend with claimants ?

b) How much time on the average does the claimant spend in the branch office ?

c) What is the average queue length ?

d) Find the average number of idle claim adjusters. **(3×20=60 Marks)**