4706

Reg. No. :

Name :

Fifth Semester B.Tech. Degree Examination, September 2014 (2008 Scheme)

(Special Supplementary)

08.502 : ADVANCED MATHEMATICS AND QUEUING MODELS (RF)

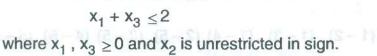
Time: 3 Hours

Max. Marks: 100

Instructions: Answer all questions of Part A and one full question each from Module I, Module II and Module III.

- 1. Define slack and surplus variables. Explain their role in LPP.
- 2. Rewrite in standard form the following LPP.

$$\begin{aligned} \text{Min } z &= x_1 + 2x_2 - 4x_3 \\ \text{Subject to} \quad & 2x_1 + 2x_2 - x_3 \leq 16 \\ & x_1 - x_2 + x_3 = 8 \\ & -x_1 + 2x_2 - x_3 \geq -7 \\ & x_1 + x_3 \leq 2 \end{aligned}$$



- 3. What are the three main phases of a project?
- 4. Define the following:
 - a) Most likely time

b) Optimistic time

c) Pessimistic time

- d) Expected time
- 5. Find the LU factorization of $\begin{bmatrix} 3 & -7 & -2 & 2 \\ -3 & 5 & 1 & 0 \\ 6 & -4 & 0 & -5 \\ -9 & 5 & -5 & 12 \end{bmatrix}$



- 6. Show that the vectors $u_1 = (1, 1, 1)$, $u_2 = (1, 2, 3)$ and $u_3 = (2, 3, 8)$ span \mathbb{R}^3 .
- 7. Find a least square solution of the system $\begin{bmatrix} -1 & 2 \\ -1 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$
- 8. Explain (M/M/I): (N/FIFO) queuing model with an example.
- 9. Find the probability that the queue size $\geq n$.
- What do you understand by a queue ? Give some important applications of queuing theory. (4x10=40 Marks)

11. Solve the following LPP.

Minimize
$$z = 2x_1 + 9x_2 + x_3$$
 such that $x_1 + 4x_2 + 2x_3 \ge 5$, $3x_1 + x_2 + 2x_3 \ge 4$, $x_1 \ge 0$, $x_2 \ge 0$, $x_3 \ge 0$.

12. A small project is composed of seven activities whose time estimates are listed in the table as follows:

Activity:	(i – j):	(1-2)	(1 - 3)	(1 - 4)	(2 - 5)	(3 - 5)	(4 - 6)	(5 - 6)
Opt. (t _o):	t	1	2	1	2	1	2	3
Most likely t _m :	t	4	2	1	5	1	5	6
Pessimistic t _p :	7	7	8	1	14	7	8	15

- a) Draw the project network.
- b) Calculate the length and variance of the critical path.
- c) What is the probability that the project will be completed atleast four weeks earlier than expected.
- d) If the project due date is 19 weeks what is the probability of meeting the due date.



Module - II

a) Solve the following equation A X = B by using LU factorization of A.

$$x + 3y + 4z = 1$$

 $-3x - 6y - 7z + 2t = -2$
 $3x + 3y - 4t = -1$
 $-5x - 3y + 2z + 9t = 2$

b) Find the dimension and construct a basis for



C(A), R(A), N(A) and N(A^T) if A =
$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

- c) Find an orthonormal basis for the subspace spanned by (1, -1, 0, 0), (0, 1, -1, 0) and (0, 0, 1, -1).
- 14. a) Find a spanning set for the null space of the matrix $A = \begin{bmatrix} 1 & -2 & 0 & 4 & 0 \\ 2 & -4 & 1 & -1 & 0 \\ 3 & -6 & 0 & 12 & 1 \end{bmatrix}$.
 - b) Find the singular value decomposition of $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 3 & 1 \end{bmatrix}$.

Module - III

- 15. a) Consider a self service store with one cashier. Assume Poisson arrivals and exponential service time. Suppose that nine customers arrive on the average every 5 minutes and the cashier can serve 10 in 5 minutes. Find:
 - i) The average number of customers queuing for service.
 - ii) Average time a customer spends in the system.
 - iii) Average time a customer waits before being served.
 - iv) The probability of having more than 10 customers in the system.
 - v) The probability that a customer has to queue for more than 2 minutes.



- b) If for a period of 2 hours in a day (8 10 AM) trains arrive at the yard every 20 minutes but the service time continues to remain 36 minutes, then calculate for this period
 - i) The probability that the yard is empty.
 - Average queue length assuming that capacity of the yard is 4 trains only.
- 16. A general insurance company has 3 claim adjusters in its branch office. People with claims against the company are found to arrive in Poisson fashion at an average rate of 20 per 8 hour day. The amount of time that an adjuster spends with a claimant is found to be negative exponential distribution with mean service time 40 minutes. Claimants are processed in the order of their appearance.
 - a) How many hours a week can an adjuster expect to spend with claimants?
 - b) How much time on the average does the claimant spend in the branch office?
 - c) What is the average queue length?
 - d) Find the average number of idle claim adjusters.

(3×20=60 Marks)